teries of crystallization. There is evidently much more in our crystallographic philosophy than we dream of, or understand. As previously set forth, it would seem that in some cases crystallic form and growth is guided and determined by interior and nucleal, or individual, rather than by external and abstract conditions.

Solid tabular hoarfrost crystals exhibiting various phases of trigonal development, are by no means rare, but it so happens that but few photographs of such have been secured for our collection. Nos. 8, 47 A, and 47 B will serve to convey an idea of their forms and structure. Lines and shadings, due to air inclusions, are prominent features of their interior structure.

Hoarfrost crystals of types HTB and HTC are usually of small size, viz, from one-twelfth to one-fourth inch in diameter.

*(14) Type HTD. Open branch or tree-like forms.*

Hoarfrost crystals grouped under this head possess an open branch-like structure, and commonly have one or more primary and many secondary rays all arranged in a very thin plane. This beautiful and frail type of hoarfrost seems to form most frequently during intense cold, when the temperature falls rapidly to zero or below. The crystals form upon and grow outward from various objects and in various situations, e. g., within barns and from the inside surfaces of barn doors, upon cobwebs and straw litter therein, and in the open upon ferns, grasses, etc., that overhang icy terraces or pools of water, the surfaces of brooks and pond ice, etc. Beautiful crystals of this variety often line open cavities in the snow or other partly closed cavities leading down to moisture, water, or wet soil. Individual crystals of this type sometimes attain to relatively large size, e. g., from 1 to 3 or more inches along their greater diameter

Photographs Nos. 11, 15, 16, 24, 158, 159, 160, and 190 portray a few of these beautiful frost creations, and also a few of the objects on which they form and which they adorn. No. 158 is a photograph of this type of frost, strung along the cobwebs hanging from a barn roof. No. 159 shows a beautiful plume-like cluster of such crystals arranged upon and around a straw stalk. No. 160 pictures them as formed in heavy white masses of clustered crystals upon the hay, barn roof, timbers, etc., of a barn loft above the stalls where cattle were kept. These, and also Nos. 24 and 158, are due to the condensation and crystallization of moisture exhaled in the breath of animals. No. 24 is an exquisitely beautiful example of this form of crystal. No. 190 is hardly less beautiful, and most remarkable because of its close resemblance to a tree.

*(15) Type HTE. Less open, branch or tree forms.*

Hoarfrost crystals of this type grow in a somewhat less open, branch-like manner than type HTD. They often consist of a large number of tiny solid tabular hexagons attached one to another, or to very short and broad branches, and arranged one outside another, all in a very thin plane. The facets of the many tiny hexagons gleam and glisten like so many diamonds and give a jewel-like appearance to the whole. These most interesting frost structures, like the preceding (type HTD), are very cold weather or zero (Fahrenheit) types. They form most frequently and in greatest number upon the bare surface of brook and river ice. They almost invariably grow upward and away from the surface of the ice. During long-continued below-zero weather large areas of river and pond ice may be thickly or completely covered with these beautiful leaf-like frost creations. Sometimes myriads of them are found clustered together into groups, like flower beds, on the surface of the ice, in the manner shown in photograph No. 170. This variety sometimes forms during a very cold night, and is found associated with other types of hoarfrost, particularly the types HTA and HTE, upon the trees and shrubs that clothe hillside and valley. Nos. 110, 111, and 208 formed in this manner upon the branches of trees, and were detached therefrom for photographic purposes.

The deposition of a heavy coat of hoarfrost of this description upon the trees in wooded regions produces a most beautiful effect, and sometimes converts a grove of trees into a fairy-land.

Photographs Nos. 13, 14, 110, 111, 168, 169, 170, 172, 173, 174, and 208 serve to reveal the forms and general outlines of this type of hoarfrost crystals. Photograph No. 174, of this series, is of more than ordinary interest. These crystals grew upward from basal points just below the streak of "Canada balsam" shown on the photograph and used by me to attach them to the glass microscope slide. At a late stage in their growth the fine frost work suddenly became of a more solid character than the portions formed before and after, as shown by the bands of larger crystals crossing the tabular structure. Atmospheric conditions were evidently such, during the formation of this more solid portion, as to cause a retardation in its rate of growth, and to favor the formation of nearly solid crystalline structures. Yet, after a time, the general conditions, such as prevailed during the formation of its basal portion, were reestablished, whereupon the crystals resumed their former and more open habits of growth.

*(16) Type HTF. Stelliform crystals.*

These form under identically the same conditions of temperature, humidity, position, etc., as those grouped under type HTB, and are often found associated with them upon the same objects. Why they fail to develop forms identical with those of type HTB can hardly be explained, except upon the supposition that nuclear differences exist, and impart their especial habits of growth to all subsequent accretions around the nuclei.

Tabular hoarfrost crystals of this description greatly resemble in all but symmetry certain solid tabular types of snow crystals. However, they rarely or never develop on a perfectly symmetrical plan as do many of the latter; commonly they develop in segmental form, because they usually crystallize upon objects in such a manner that but three or four of the six corners of the hexagon have an opportunity of growing outward from the nucleus.

*[To be continued.]*

## COTTIER'S RESISTANCE OF ELASTIC FLUIDS.

The pressure of the wind for any given velocity, or the resistance of the air to a moving body, is one of the fundamental questions in the physics of the atmosphere. The subject has been treated experimentally by practical engineers and laboratory physicists for three centuries past; but their measurements have mostly served to show how little we understand the flow of air around and behind an obstacle. The physicist needs the guiding hand of a master in analytical mechanics. Summaries of the present state of experimental knowledge of the subject were attempted by myself in my lectures of 1882,[1] and in my Treatise on Meteorological Apparatus and Methods[2]; in a memoir by Capt. W. H. Bixby, U. S. Army Engineer Corps, in 1891; in Schreiber's Studien über Luftbewegungen, 1898; and in Bigelow's "Relations between wind velocities and atmospheric pressures".[3] The fundamental hydrodynamic formulas are given by Lamb, Basset, Love, Helmholtz, Wien, Auerbach, Saint Venant, Boussinesq, and other writers on hydrodynamics.

The late J. G. C. Cottier, author of the memoir on "The equations of hydrodynamics in a form suitable for application to problems connected with the movements of the earth's atmosphere",[4] left several excellent manuscripts bearing on

[1] Ann. Rep. C. S. O., 1882, pt. 1, p 98.
[2] Ann. Rep. C. S. O., 1887, pt. 2.
[3] Monthly Weather Review, October, 1906, (vol. XXXIV, p. 470).
[4] Monthly Weather Review for July, 1897, (vol. XXV, p. 296).

**atmospheric phenomena, one of which we now publish by the kind permission of the President of Columbia University and of Prof. R. S. Woodward, the literary executor of Mr. Cottier. This short paper by Mr. Cottier is especially valuable as indicating the hypotheses or ideas on which his predecessors have based their researches.**

By his mental grasp of the complex movements of the air near any obstacle, and his ability to express in rigorous formulas the mechanical reactions that result therefrom, Mr. Cottier gave promise of becoming a remarkably able investigator, and his untimely death was undoubtedly a great loss to meteorology.—*C. A.*

## A SUMMARY OF THE HISTORY OF THE RESISTANCE OF ELASTIC FLUIDS.

By JOSEPH G. C. COTTIER. Dated Columbia University, New York, N. Y., April 27, 1896.

By elastic fluids are understood such fluids as air and other gases, and it is intended to restrict the discussion to such velocities only as are small in comparison to the velocity of sound in the gas. With the exception of ballistic problems and the motion of gases escaping freely from an orifice, almost all ordinary questions fall within this restriction.

Keeping the velocity within these bounds introduces a great simplification in the analysis, for then compressible fluids may, without gross error, be treated as incompressible.

Many writers claim to have discovered that the resistance offered to a moving body by a fluid at rest is not equal to the pressure exerted by a moving fluid on a solid at rest; but the experiments upon which this deduction is based are so unsatisfactory, and the statement itself so improbable, that no allowance has been made in the following essay for such a phenomenon.

The original papers of the writers referred to have been consulted whenever possible; otherwise the authority is given in a footnote.

The history of air resistance may be said to date from the time of Galileo. In his "Discorsi", 1638, he showed that, in consequence of the laws of falling bodies, discovered by him in 1602, the path of a projectile must be parabolic, *if not affected by the resistance of the air;* but his disciples disregarded this injunction, reasoning that a fluid as light as air could not appreciably affect the motion of so heavy a body as a projectile.[2]

In 1668–69 a committee of the Royal Academy of Sciences of Paris, consisting of Messrs. Huygens, Mariotte, Picard, and Cassini, made a series of experiments on bodies immersed in currents of water, and from these Huygens deduced the law that the resistance is proportional to the square of the velocity, and also that the pressure on a plane surface is the same as that due to a statical column of the fluid, of height equal to the head due to velocity.

According to Saint Venant,[3] Pardies showed as early as 1671 that for ships' sails the pressure should be proportional to the $\sin^2 \alpha$, where $\alpha$ has that meaning which will be assigned to it thruout this paper; i. e., it is the angle between the direction of the motion and the plane of the surface, or the complement of the "angle of incidence".

Certain it is, however, that in his "Traite du Mouvement des Eaux", published posthumously in 1686, Mariotte determined the law that resistance is proportional to the square of the velocity, from considerations based on the impact of the molecules of the fluid on the body; and that in the same paper he deduced geometrically the law that the pressure is proportional to the $\sin^2 \alpha$.

Mariotte died in 1684, and as Newton's "Principia" did not appear until 1687, the credit for the famous laws,

$P$ is proportional to $(\text{Vel.})^2$

and

$P$ is proportional to $\sin^2 \alpha$,

which occur implicitly in Propositions 34 and 35, Book II, of the "Principia", belongs not to Newton, but to Huygens, and to Pardies and Mariotte, respectively.

By some experiments on falling bodies Newton was made aware of the fact that the Huygenian theory of hydrodynamical pressure was not in accordance with practise, and in Proposition 36, Book II, by a process that is unsatisfactory in the extreme, he corrected it so as to give a resultant pressure equal to *one-half* the pressure of a statical column of the fluid of head due to velocity, a result which agreed better with experiment than the first-named law. However, the geometers did not take kindly to Newton's amended theory, but clung to the original Huygenian law.

S'Gravesande, in his work on natural philosophy, 1725, was the first to disagree with Mariotte's or Pardies's law,

$P$ is proportional to $\sin^2 \alpha$,

and to offer the law

$P$ is proportional to $\sin \alpha$.

For small values of $\alpha$ this gives a better result than the former, and was deduced from the consideration that a fluid is not constructed of independent particles, but of a substance that has the property of exerting the same normal pressure in all directions.

Daniel Bernoulli, in 1727, proposed a theory which would have given hydrodynamical pressure equal in amount to the hydrostatical pressure of a column of water of twice the head due to the velocity, but he abandoned this later; and in a memoir published in 1736, making for the first time a distinction between the pressure exerted by an infinite fluid on a body and that due to an isolated jet, he derived that method of treating the latter which has survived to the present day.[4]

Maclaurin's contributions (1742) to this branch of science appear to be confined to the formula for the angle of maximum effort of windmill sails, when $P$ is proportional to $\sin^2 \alpha$.

He found

$$\tan \alpha = \tfrac{3}{2}\frac{v}{V} + \sqrt{2 + \tfrac{9}{4}\frac{v^2}{V^2}}$$

where $v$ equals velocity of the vane, and $V$ that of the wind (at right-angles to the first). This is of importance as the first correction to the error in Mariotte's (1686) and Parent's (1704) analysis, which upon the same hypothesis gave $\alpha$ the constant value $55° \pm$, for the effect of the motion of the vane had been neglected.

Robins made a distinct step in advance when in his "New Principles of Gunnery", 1742, he described his apparatus for experimental determination of the resistance of the air, and gave the results of a few tests. This apparatus, the first of its kind, continued much in favor among the later English experimenters. The bodies under observation were fixt at the end of a horizontal arm, rotating about a vertical axis; a falling weight gave the power necessary to keep the arm in motion, and the revolving body itself served the purpose of a governor.

Robins's work was translated into French and annotated by Leonhard Euler. In a note the commentator attempted to obtain a mathematical explanation for the phenomena by summing the components in the direction of motion of the deviating forces necessary to deflect the stream lines from their originally straight path to their disturbed condition. Unfortunately, for a frictionless fluid, such a method gives zero for result, unless the posterior three-quarters of each filament be

[1] Submitted in partial fulfilment of the requirements for the degree of Master of Arts.

[2] Rühlmann, Hydromechanik, second edition, 1880.

[3] B. de Saint Venant, Resistance des Fluides. Published posthumously in the Memoires of the Paris Academy, 1888.

[4] B. de Saint Venant, op. cit.